

Probabilistic Population Projections for Countries with Generalized HIV Epidemics*

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Abstract

Population projections tell us about the future size and age composition of a population. Governments, international organizations, and researchers use these projections for planning, monitoring development goals, and research purposes. Typically, projections are done deterministically, but there is extensive interest in probabilistic projections. Where probabilistic projections are available, they are limited to countries without substantial levels of HIV prevalence because current mortality projection methodology is inapplicable due to the high number of adult deaths resulting from a generalized HIV epidemic. We propose a method for making probabilistic population projections for countries with generalized HIV epidemics by incorporating the future trajectory of the epidemic in the mortality component of the projection. Our method takes into account uncertainty about future levels of mortality and fertility, the major drivers of population change, as well as uncertainty about the trajectory of HIV prevalence.

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1 Introduction

Population projections tell us about the future size and age composition of a population. Governments, international organizations, and researchers use these projections for planning future allocation of societal resources, monitoring development goals, and in the course of other lines of social and health research. The United Nations Population Division (UNPD) produces such projections broken down by age and sex and publishes them in the biennial series *World Population Prospects* (WPP).

Until the 2010 revision (United Nations, Department of Economic and Social Affairs, Population Division, 2011a), the UN projections, like most other projections, were produced deterministically using the standard cohort component method (Whelpton, 1936; Leslie, 1945; Preston et al., 2001). The cohort component projection method is based on the basic demographic balancing equation where the population of a country at time $t + 1$ is equal to the population at time t plus births and immigrants and minus deaths and emigrants. The deterministic nature of this method yields a single projected value for the quantity of interest, that is conditional on assumed levels of future fertility, mortality and international migration. However, because the future is inherently uncertain, it is desirable to be able to quantify uncertainty about these values. Probabilistic projections yield probability distributions about the quantity of interest and can provide useful information for decision makers and researchers. To this end, the recently released WPP 2012 revision (United Nations, Department of Economic and Social Affairs, Population Division, 2013a) incorporates probabilistic projections of fertility and mortality and Raftery et al. (2012) have produced probabilistic population projections for nearly all countries of the world.

Excluded from these recent probabilistic projections are countries with generalized HIV epidemics, defined by having prevalence above 1% and not being concentrated in specific subpopulations. The mortality component of the current UN projection methodology for countries without generalized epidemics is not suitable for high-HIV prevalence countries because the age profile of incidence results in a large number of deaths among young adults and the middle-aged resulting in a particular age pattern of mortality rates. For these countries, it is necessary to explicitly incorporate the future trajectory of the epidemic in the projection model.

The 2012 Revision incorporates the demographic impact of AIDS in a deterministic projection for 39 countries where HIV prevalence is above 2 per cent between 1980 and 2011 in the general population (United Nations, Department of Economic and Social Affairs, Population Division, 2013c, see table v.2 for list of countries). The UNPD uses a multi-compartment model to project mortality due to HIV/AIDS and mortality among the non-infected population separately. For HIV/AIDS mortality, the UNPD models the course of the epidemic and projects yearly incidence of HIV prevalence with the model developed by the UNAIDS Reference Group on Estimates, Modelling and Projections, implemented in the latest release

of the Spectrum/Estimates and Projection Package (EPP) software (Stanecki et al., 2012; Stover et al., 2012).¹ In the multi-compartment model approach, compartments of HIV⁻ and HIV⁺ people are generated and different age-specific mortality rates are applied for projecting the two groups.

This process requires a large number of parameters (e.g. incidence rates and sex-specific prevalence) for which little data exist to estimate them. Furthermore, this approach is somewhat impractical because the UNPD is reliant on an external firm to produce key inputs to the model. To this end, we seek a parsimonious model that uses the standard UNPD population projection infrastructure but can be executed with simple, in-house produced inputs. In this paper, we present a probabilistic approach that meets these criteria for making population projections for the 40 countries with generalized HIV epidemics listed in table 1.

Table 1: Countries affected by generalized HIV/AIDS epidemics. These countries had > 1% prevalence in the general population at any point during 1980-2010 and no concentration in high-risk subgroups.

	Africa	Caribbean/Latin America
Angola	Kenya	Bahamas
Benin	Lesotho	Belize
Botswana	Liberia	Guyana
Burkina Faso	Malawi	Haiti
Burundi	Mali	Jamaica
Cameroon	Mozambique	
Central African Republic	Namibia	
Chad	Nigeria	
Congo	Rwanda	
Cote d'Ivoire	Sierra Leone	
Djibouti	South Africa	
Equatorial Guinea	Swaziland	
Ethiopia	Togo	
Gabon	Uganda	
Gambia, The	United Republic of Tanzania	
Ghana	Zambia	
Guinea	Zimbabwe	
Guinea-Bissau		

¹Public versions of Spectrum are available at: <http://www.futuresinstitute.org/pages/resources.aspx>.

2 Methods and Data

Population projection involves combining future values of age-specific fertility, age- and sex-specific mortality, and international migration rates, in this case using the standard cohort component model. To make probabilistic projections for high-HIV prevalence countries, we follow the procedure described by Raftery et al. (2012) for making projections of fertility and migration in countries without significant levels of HIV prevalence, but we modify the mortality component to account for HIV/AIDS mortality.

Raftery et al. (2012) simulate a large number of trajectories of the Total Fertility Rate (TFR) using the Bayesian hierarchical model of Alkema et al. (2011). The projected TFRs are then converted to age-specific rates using model fertility patterns. For mortality, Raftery et al. (2012) simulate an equal number of trajectories of period life expectancy at birth (e_0) for females using the model of Raftery et al. (2013). Male life expectancies are conditional on the female e_0 and are derived from a model that predicts the gap in male and female life expectancy (Lalic and Raftery, 2014). These life expectancy projections are converted to age-specific mortality rates using a variant of the Lee-Carter method. The fertility, mortality, and migration trajectories are then converted to a future trajectory of age- and sex-specific population values using the cohort component model (Preston et al., 2001, ch. 6).

In our application, the projection of fertility² remains the same along with the use of the cohort component method to combine these trajectories. However, HIV prevalence is now included as a predictor when modeling the future trajectory of life expectancy and in converting the projected e_0 into age-specific mortality rates. A brief description of the methods for projecting e_0 and HIV prevalence as well as converting those quantities into age-specific mortality rates follows.

2.1 Projecting HIV prevalence

To make probabilistic projections of e_0 for countries with generalized epidemics and to convert those projections into age-specific mortality rates, we first need projections of HIV prevalence. We use an R version of the UNAIDS Estimates and Projections Package (EPP) (Brown et al., 2010; Ghys and Garnett, 2010; Alkema et al., 2007; Raftery and Bao, 2010) to make probabilistic projections of HIV prevalence up to 2100.

EPP works well to project HIV prevalence into the not too distant future, but assumptions were imposed on two of the EPP model parameters to make projections out to 2100. For most countries, the model is fitted assuming that the relevant parameters have remained constant in the past. Beginning in the start year of the projection, the parameter f_{XN} , which

²Fertility is projected using the `bayesTFR` package (Ševčíková et al., 2011) in the statistical analysis software, R.

reflects the rate of recruitment of new individuals into the high-risk or susceptible group, is projected to decline by half every 20 years. The parameter r , which represents the force of infection, is projected to decline by half every 30 years. The reduction in r reflects the assumption that changes in behavior among those subject to the risk of infection and increases in access to treatment for infected individuals will reduce the chances of HIV transmission.

The trajectories were then calibrated by the HIV prevalence projections of Hontelez et al. (2012), who make deterministic projections up to 2040. They used STDSIM, a stochastic microsimulation model, which is a plausible method to predict future trends in HIV prevalence. We recorded the difference on the probit scale between the EPP median and the Hontelez et al. projection in each year from 2011 to 2040 so that we shift all the trajectories by the difference for calibration. For projection past 2040, we assumed that the difference in probit scale remains constant from 2040-2100.

2.2 Projecting life expectancy at birth, e_0

Because a generalized HIV epidemic can have a considerable depressing effect on life expectancy at birth in a short time (Blacker, 2004; Ngom and Clark, 2003; Obermeyer et al., 2010; Poit et al., 2001; Timaeus and Jasseh, 2004; Sharrow et al., 2013a), a model that reflects the impact of HIV prevalence and antiretroviral therapy (ART) coverage is necessary to make appropriate projections of life expectancy in generalized epidemics. The model for projecting life expectancy, equation (1), predicts the 5-year change (gains) in total (non-sex-specific) life expectancy at birth from time $t - 5$ to t as a function of two covariates: a double logistic curve³ based on the life expectancy at time $t - 5$ and HIV prevalence at time t multiplied by the proportion of infected individuals not receiving ART:

$$\Delta e_{0,c,t} = \beta_0 + \beta_1 e_{0,c,t-5}^{DL} + \beta_2 nonART_{c,t}, \quad (1)$$

where $\Delta e_{0,c,t}$ is the change in life expectancy for country c at time t , $e_{0,c,t-5}^{DL}$ is the double logistic fitted life expectancy from time $t - 5$, and $nonART_{c,t}$ is HIV prevalence at time t times the proportion of seropositive individuals not receiving ART at time t .

The model (1) is estimated using 5-year life expectancy estimates from 1990-2010 for the 40 countries experiencing a generalized HIV epidemic obtained from WPP 2012⁴. We use median HIV prevalence for 1990-2010 as estimated by the EPP program. We use estimates of ART coverage (% of infected individuals receiving ART) for the period 1990-2010 obtained from the UN Population Division internal tabulations used to produce WPP 2010 (United Nations, Department of Economic and Social Affairs, Population Division, 2011b). To make this model compatible with the model for countries without generalized epidemics, we set β_1 , the coefficient for the double logistic curve, to 1 when we fit the model using regression.

³We use the same parameters for the double logistic curve as were used in WPP 2012 (United Nations, Department of Economic and Social Affairs, Population Division, 2013a).

⁴These estimates are available in the R package `wpp2012` (Ševčíková et al., 2013).

To project life expectancy to 2100, we input each trajectory of HIV prevalence and life expectancy period by period into the model starting from 2010.

2.3 Converting e_0 and HIV prevalence projections to age-specific mortality rates

Once we have obtained probabilistic projections of life expectancy and HIV prevalence, we need to map those quantities onto a set of age-specific mortality rates that can be combined with age-specific fertility rates and net migration using the cohort component method. In the WPP 2012 Revision, for countries without high HIV prevalence, e_0 projections are converted to age-specific mortality rates using model mortality patterns (United Nations, Department of Economic and Social Affairs, Population Division, 2013b, p. 34), but these patterns are unable to replicate the particular age pattern of mortality resulting from large scale HIV epidemics (United Nations, Department of Economic and Social Affairs, Population Division, 2013b, p. 35) and they have no relationship to HIV prevalence.

To convert the life expectancy and HIV prevalence projections to age-specific mortality rates we use a model by Sharrow et al. (2013b), shown in equation 2. This model can reproduce the characteristic age pattern of mortality associated with generalized epidemics, i.e. an accentuated adult mortality hump concentrated at ages 30 to 45. The model represents the age pattern of mortality rates as a weighted sum of three age-varying components. The components, $b_{i,x}$ in equation 2, are derived from a Singular Value Decomposition of the matrix of observed mortality rates and the weights, $\omega_{i,\ell}$, are modeled as a function of HIV prevalence and life expectancy at birth. We refer to this model as ‘HIV MLT’ as it essentially gives HIV prevalence-calibrated model mortality patterns.

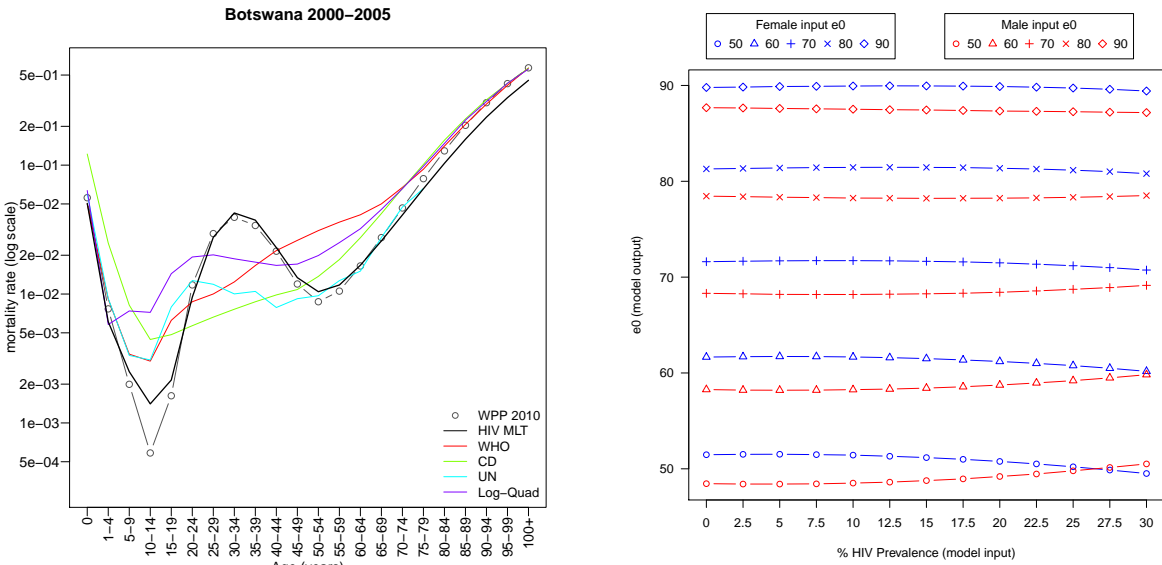
$$\ln({}_n m_{x,\ell}) = c_\ell + \sum_{i=1}^3 \omega_{i,\ell} b_{i,x} + \varepsilon_{x,\ell}, \quad (2)$$

where ${}_n m_{x,\ell}$ is the period age-specific mortality rate from age x to age $x + n$ for life table ℓ , c_ℓ is a constant specific to life table ℓ , $b_{i,x}$ is the value of the i th component for age x , $\omega_{i,\ell}$ is the weight of the i th component for life table ℓ , and $\varepsilon_{x,\ell} \stackrel{\text{iid}}{\sim} N(0, \sigma^2)$ is the error term.

Figure 1a plots the fit from the HIV MLT model and four existing model life table systems for Botswana females 2000-05. HIV prevalence during this period was roughly 26 percent in Botswana resulting in a large adult mortality hump, which figure 1a demonstrates is fully captured only by the HIV MLT model. All other systems tend to produce high, flat patterns of mortality rates that match the level of mortality but miss the age-specific rates.

This model takes e_0 and HIV prevalence as inputs and produces a set of age-specific mortality rates that reflect those two inputs. HIV MLT is designed to produce a set of age-specific mortality rates that yield an output life expectancy matching the input life expectancy. The

HIV MLT model was originally calibrated with sex-specific e_0 (Sharro et al., 2013b), but for the present purpose we have re-calibrated the model with the total (non-sex-specific) e_0 because that is what is projected by the model described in section 2.2. To maintain the gap between male and female e_0 , the re-calibrated model produces complete sets of male and female age-specific mortality rates simultaneously and matches the input e_0 to the combined life expectancy derived from the output male and female mortality rates by adjusting the intercept, c_ℓ in equation 2. Figure 1b plots the sex-specific output e_0 from the HIV MLT model while varying the two input parameters: non-sex-specific e_0 and HIV prevalence. The gap in life expectancy is maintained except at very low life expectancies coupled with very high HIV prevalence, which is expected due to the sex-specific age pattern of deaths resulting from very high HIV prevalence.



(a) Fit of HIV MLT model for Botswana females 2000-05 (b) Sex-specific e_0 output for HIV MLT model at varying e_0 and prevalence inputs

Figure 1: Output from HIV MLT model (Sharro et al., 2013b). (a) Fit of HIV MLT model for Botswana females 2000-05. HIV MLT model shown with black line. For comparison fits from the WHO modified logit model (Murray et al., 2003) [red solid line], Coale and Demeny model life tables (Coale and Demeny, 1966; Coale et al., 1983) [green solid line], UN model life tables for developing countries (United Nations. Department of International Economic and Social Affairs, 1982) [teal solid line], and the Log-Quad model (Wilmoth et al., 2012) [purple solid line] are also shown. (b) shows sex-specific output e_0 while varying the two HIV MLT model inputs: non-sex-specific e_0 and HIV prevalence.

Finally, this model is calibrated with five-year age-specific mortality rates obtained from WPP 2012 from 1990-2010 for the 40 countries experiencing a generalized epidemic. HIV prevalence for calibrating this model is the same as for the model for projecting e_0 (see

section 2.2). Because the e_0 and prevalence projections are probabilistic, we get a large set of probabilistic mortality age pattern projections.

2.4 Making full probabilistic population projections

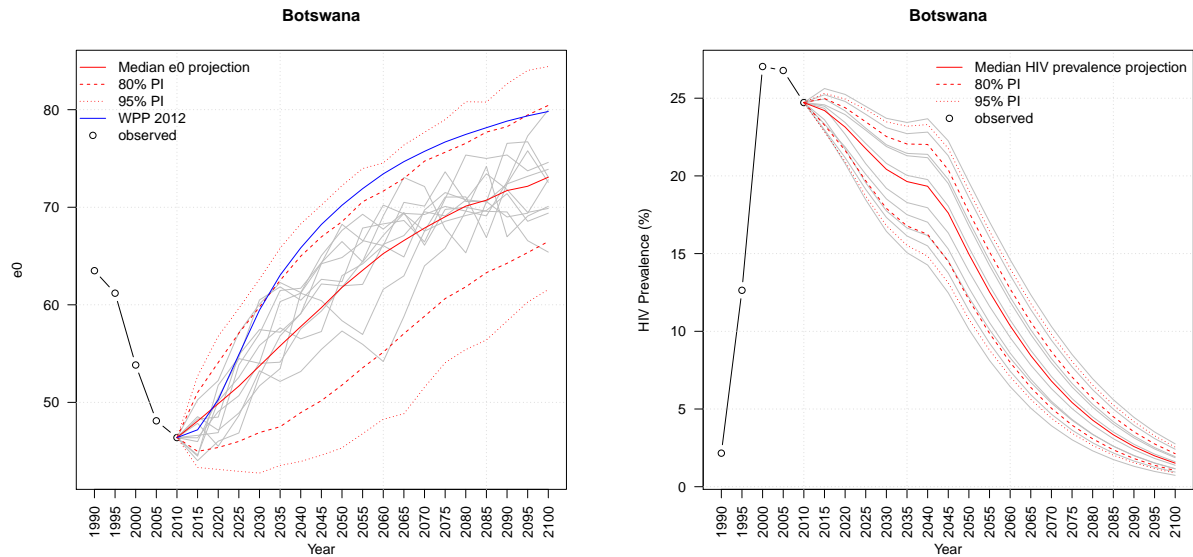
We use the `bayesPop` software (Ševčíková and Raftery, 2013), which combines the fertility and mortality projections using the cohort component method, to make full population projections. `bayesPop` uses the method described by Raftery et al. (2012) to produce mortality projections, so the package functions were altered to include the mortality methodology described above.

3 Results

We show results here for four countries: Botswana, Lesotho, Mozambique and Ghana. These countries were chosen to represent different levels of HIV prevalence. In 2010 Botswana and Lesotho represent the largest epidemics with HIV prevalences of roughly 25 percent; Mozambique represents a smaller but still substantial epidemic (HIV prevalence in 2010 \approx 15 percent); while Ghana has a small generalized epidemic (HIV prevalence in 2010 \approx 1.5 percent). Results for Botswana are shown in figure 3, while results for the other three countries are in figures 6-8 in appendix A.

Botswana is experiencing one of the largest HIV epidemics in the world. Figure 2 shows the probabilistic projection of HIV prevalence and life expectancy at birth for Botswana 2010-2100 (past observations are shown by black circles, median projection and prediction intervals are shown in red, and the e_0 derived from the WPP 2012 Revision mortality projections is shown by a blue line). HIV prevalence in Botswana is projected to remain high dropping from about 24 percent to roughly 13 percent by 2050. Although declining, these high prevalence rates contribute to slow growth in life expectancy reaching just above 60 years by 2050 from less than 50 years in 2010. Compared to the WPP 2012 Revision projected life expectancies, our median projection shows sustained lower life expectancy of roughly 5-7 years over the entire projection period.

Turning to panel a (*top left*) of figure 3, which plots the total population projection for Botswana, we project an increase in population until about 2080 when the total population begins to decline (observed data: black circles; median probabilistic projection: solid red line, 80% predictive interval: dashed red lines; 95% predictive interval: dotted red lines; WPP 2012 Revision population projection: solid blue line). Also in figure 3a note the increasing uncertainty as the projection reaches farther into the future, a feature of the projection for all countries. The WPP 2012 Revision (solid blue line) also shows sustained population growth followed by a mild reversal in that trend close to 2100, but our projection predicts fewer people in the total population over the entire projection horizon.



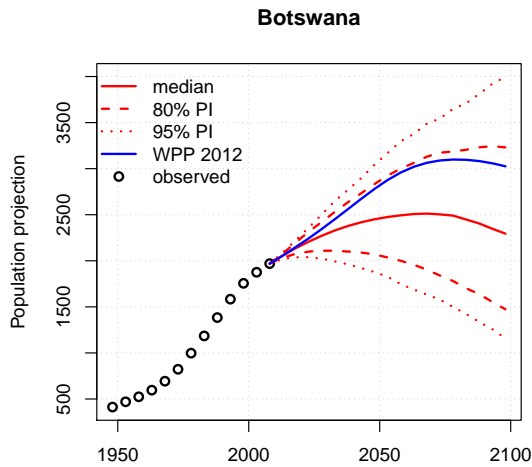
(a) Probabilistic Life Expectancy Projection for Botswana

(b) Probabilistic HIV Prevalence Projection for Botswana

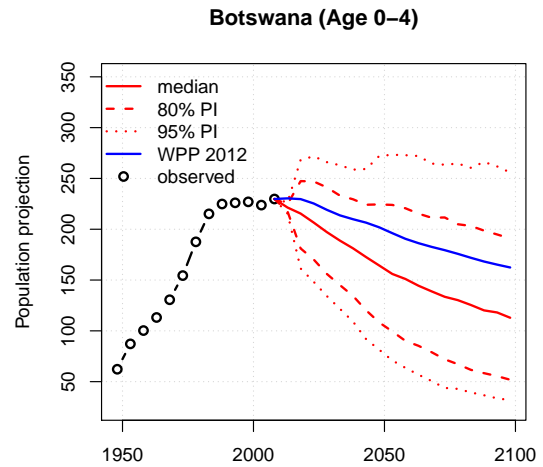
Figure 2: Probabilistic life expectancy and HIV prevalence projections for Botswana 2010-2100. median of probabilistic projection: solid, red line; 80% predictive interval: dotted, red line; 95% predictive interval: dashed, red line; WPP 2012 Revision projection: solid, blue line; observed: black circles. The gray lines in these figures are a random sample of ten trajectories from the final sample of 1,000 trajectories from the posterior distribution.

Figure 4 shows the proportional difference between the total population in 2050 from our projection and the WPP 2012 Revision relative to the 2012 Revision for each of the 40 countries under study here ($[\text{our projection}_{2050} - WPP_{2050}]/WPP_{2050}$). Botswana has the largest difference from WPP 2012 in 2050 with about 12 percent fewer people in the total population compared to WPP 2012. This difference arises from our treatment of mortality in the projection. In addition to projecting lower life expectancy than WPP 2012 (see figure 2a), our method also produces relatively high age-specific mortality rates at ages 25-45 consistent with the mortality generated under high HIV prevalence. High mortality during the reproductive years for women (see figure 3d) results in fewer women alive during the reproductive years and thus fewer births.

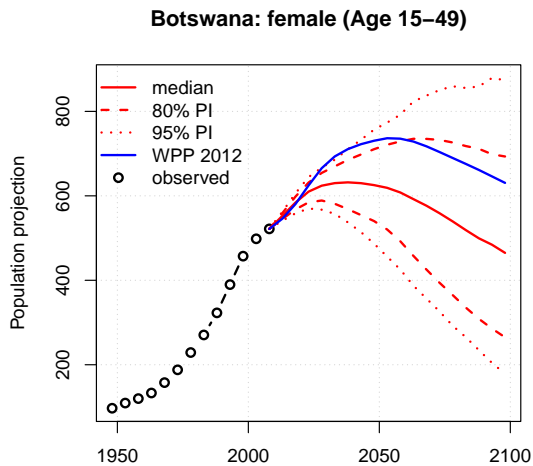
These effects are shown in figures 3b-3d. Figure 3c shows the probabilistic population projection for women aged 15-49. We project a decline in the number of women in this age category beginning prior to 2050 and figure 3b shows a shrinking population under age five over the entire projection horizon. The effect of high mortality in the reproductive adult years, especially for women, on future population size reverberates for a number of years as smaller cohorts are born in each projected period, resulting in smaller total population



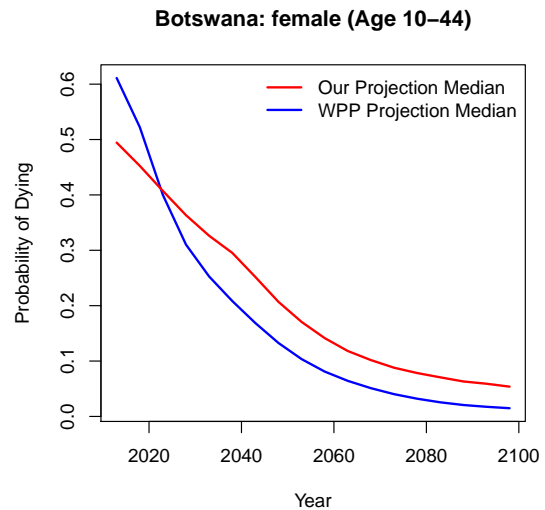
(a) Population Projection for Botswana, total population



(b) Population Projection for Botswana, age 0-4



(c) Population Projection for Botswana, female age 15-49



(d) Projection of the probability a female at age 10 will die before her 45th birthday

Figure 3: Probabilistic population projections for Botswana 2010-2100. For panels a, b, and c: observed data: black circles; median probabilistic projection: solid red line, 80% predictive interval: dashed red lines; 95% predictive interval: dotted red lines; WPP 2012 Revision population projection: solid blue line.

Population Projection Median Comparison in 2050

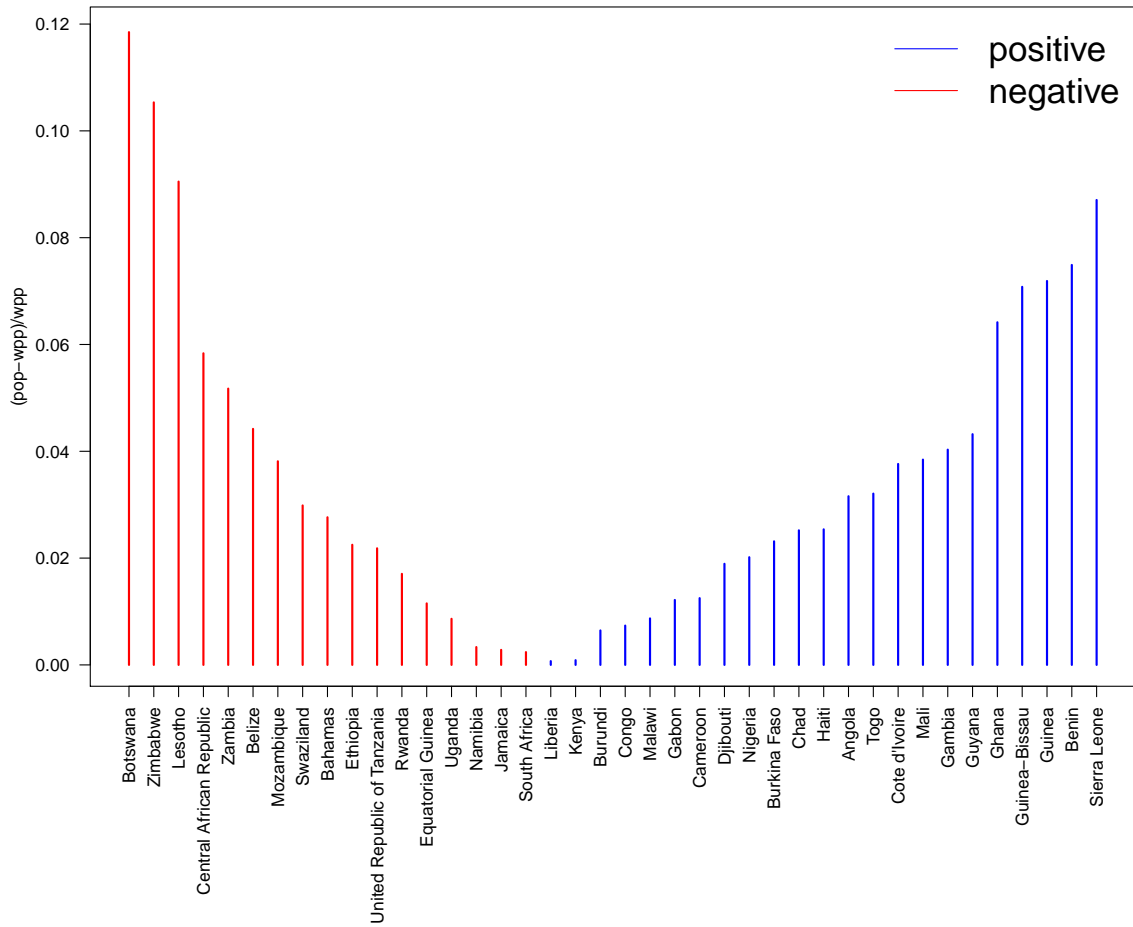


Figure 4: Comparison between our median population projection and WPP 2012 median projection at 2050. Each vertical line shows the difference between our median projection for the year 2050 and the WPP 2012 Revision median projection for 2050 relative to the WPP projection. Projections with a positive difference (i.e. our projection is greater) are shown in blue. Projections with a negative difference (i.e. WPP 2012 Revision projection is greater) are shown in red.

compared to WPP 2012. Likewise, combined with declining fertility, the effect of high adult mortality yields an eventual reversal in population growth for Botswana.

Similar conditions exist for all the countries with the top five negative proportional differences (all with differences of greater than five percent) shown in figure 4. All five of these countries have large scale HIV epidemics (> 10% prevalence) again reducing the number of women of reproductive age resulting in smaller birth cohorts. Lesotho is projected to have

about eight percent fewer people in the total population in 2050 compared to the WPP 2012 Revision. HIV prevalence is also projected to decline from about 24 percent to 18 percent between 2010 and 2050. We project higher probabilities of death for women of reproductive age compared to WPP 2012 (figure 6d) resulting in a likely decreasing number of women of reproductive age past 2025 (figure 6c) and consequently smaller birth cohorts over the projection horizon (figure 6b).

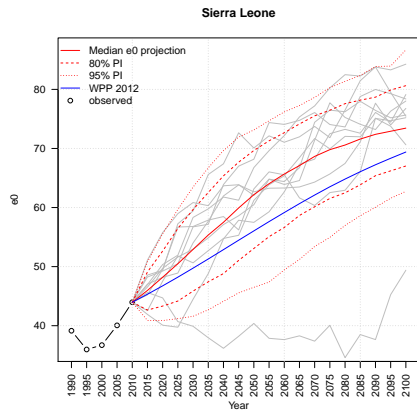
For countries with smaller HIV epidemics, the reduction in the number of women of reproductive age is less severe and thus the difference between our projections and WPP 2012 revision tends to be smaller. Mozambique is projected to have just under 10 percent prevalence (median projection) by 2050, down from around 15 percent in 2010. Figure 7a shows that our median projection of the total population is similar to the WPP 2012 projection, but we again project a smaller total population (4 percent difference in 2050, see figure 4). The smaller differences from WPP 2012 can also be seen in figures 7b and 7c showing the population projections for under age five and women age 15-49 respectively.

For Ghana, where HIV prevalence is projected to decrease from about 1.5 percent in 2010 to under one percent by 2050, the difference from WPP 2012 is in the opposite direction. We project about six percent more people in the total population by 2050 compared to WPP 2012. The much lower rates of HIV prevalence have a far less extreme depressing effect on total population in Ghana in the long run as evidenced by figures 8b and 8c. In addition to relatively little effect from HIV on the age-specific mortality rates, compared to WPP 2012, we project consistently higher life expectancy over the projection period for Ghana along with the other top five positive difference countries compared to WPP 2012 (see figure 4).

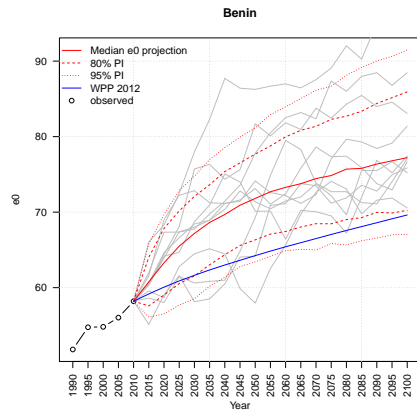
Figure 5 plots the probabilistic projections of life expectancy for the six countries with the largest percentage differences in projected population, as shown in figure 4. It also depicts the life expectancy derived from the WPP 2012 Revision mortality projections for these countries (past observation is shown in black circles, median projection and prediction intervals shown in red, and the e_0 derived from the WPP 2012 Revision mortality projections is also shown for comparison with a blue line). As this figure shows, a large portion of the difference between the WPP 2012 Revision total population projections and our median projections arises from differences in the projections of life expectancy, since these countries have comparatively small HIV epidemics.

4 Discussion

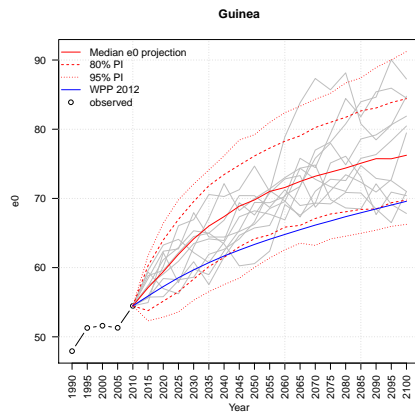
We have presented a method for making probabilistic population projections for countries with a generalized HIV epidemic. We accomplish this by following the Bayesian probabilistic projection method described by Raftery et al. (2012) for fertility and migration, but because of the singular nature of mortality in generalized HIV epidemics, we modify the mortality component of the projection to incorporate the future trajectory of the epidemic in terms of



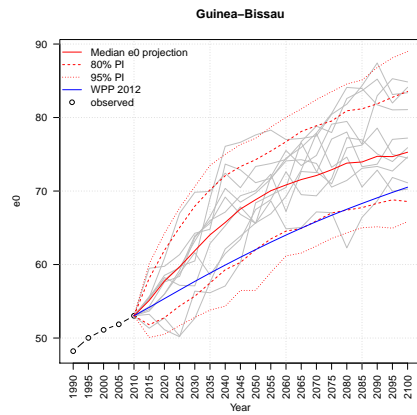
(a) e_0 projection for Sierra Leone



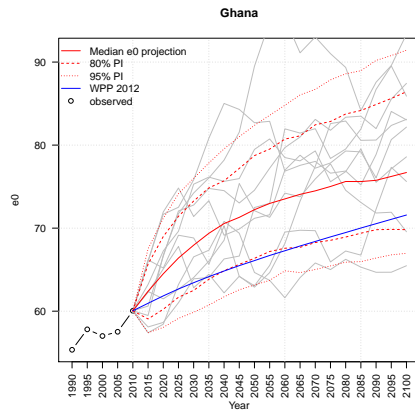
(b) e_0 projection for Benin



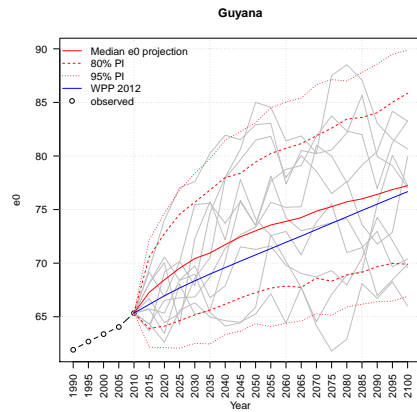
(c) e_0 projection for Guinea



(d) e_0 projection for Guinea-Bissau



(e) e_0 projection for Ghana



(f) e_0 projection for Guyana

Figure 5: Probabilistic life expectancy projections for six countries 2010-2100. median of probabilistic projection: solid, red line; 80% predictive interval: dotted, red line; 95% predictive interval: dashed, red line; WPP 2012 Revision projection: solid, blue line; observed: black circles. The gray lines in these figures are a random sample of ten trajectories from the final sample of 1,000 trajectories from the posterior distribution.

HIV prevalence and ART coverage. The probabilistic fertility and mortality projections and migration are combined using the cohort component method of projection. These projections are potentially useful to researchers and policy makers as this method provides a predictive distribution for population quantities of interest such as total population, life expectancy, and support ratios into the future. Our method takes into account uncertainty about future levels of mortality and fertility, the major drivers of population change, as well as uncertainty about the trajectory of HIV prevalence.

Results from the projections described here show that by 2050 and beyond, we project smaller total populations for 17 of the 40 countries under study here compared to WPP 2012. Many of the countries with the largest differences in projected population compared to WPP 2012 have large scale HIV epidemics. For these countries, we tend to project lower total life expectancy over the course of the projection period. Combined with projected high HIV prevalence, the lower life expectancies result in high age-specific mortality rates during the younger adult years, and thus fewer women of reproductive age and smaller birth cohorts. Projected into the future, these trends lead to smaller total population projections compared to WPP 2012. Likewise, coupled with declining fertility, high mortality rates resulting from HIV/AIDS-related deaths produce a reversal in population growth by 2100 for some countries with very large epidemics.

Although the method presented here for mortality and elsewhere for fertility takes into account uncertainty about future levels of fertility and mortality, it does not include uncertainty about international migration in the future, which can be a large source of forecast errors in the short run (Raftery et al., 2012). Likewise, the life expectancy projection model and the model used to convert e_0 projections to age-specific mortality rates are calibrated with results from WPP 2012 (which are themselves modeled), so they reproduce only the variability in the quantities of interest contained in the WPP results. To the extent that the WPP 2012 data and results used to calibrate these models reflect the empirical reality, the models we present here should as well. Finally, as in Raftery et al. (2012), this method does not take into account random variation in the number of birth or deaths.

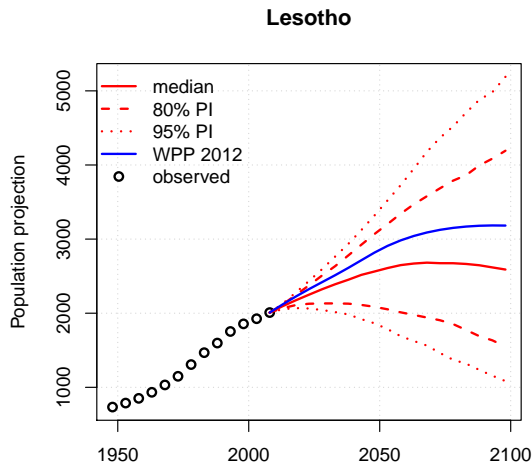
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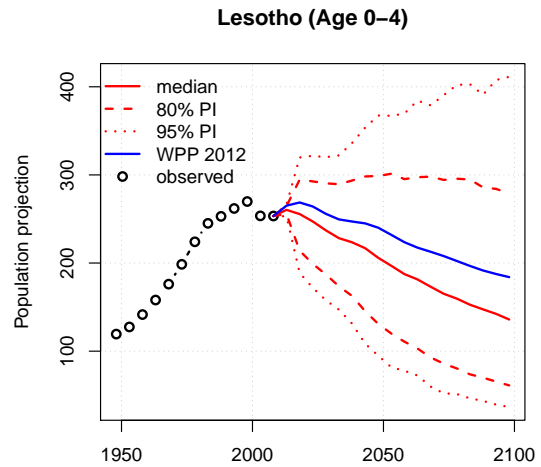
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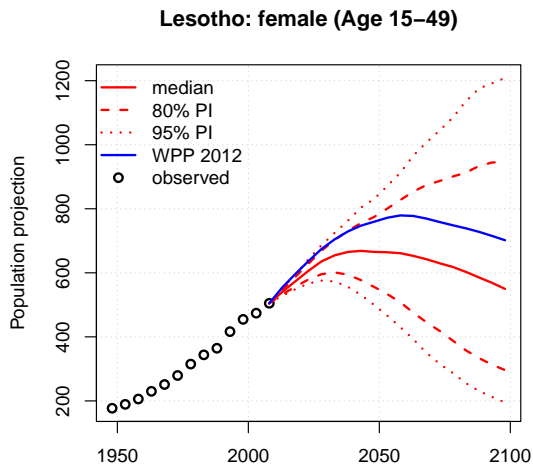
A Probabilistic Population Projections for selected countries



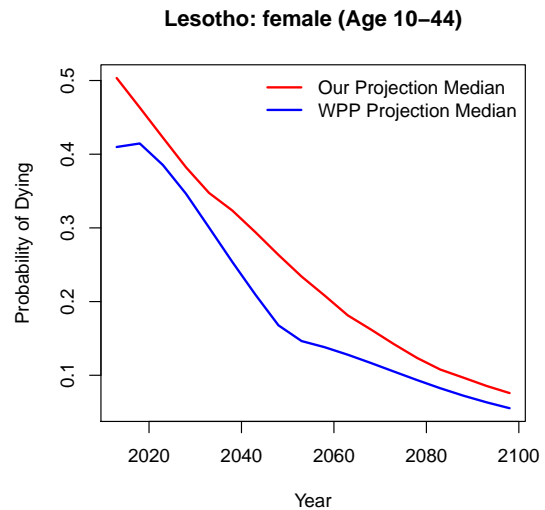
(a) Population Projection for Lesotho, total population



(b) Population Projection for Lesotho, age 0-4

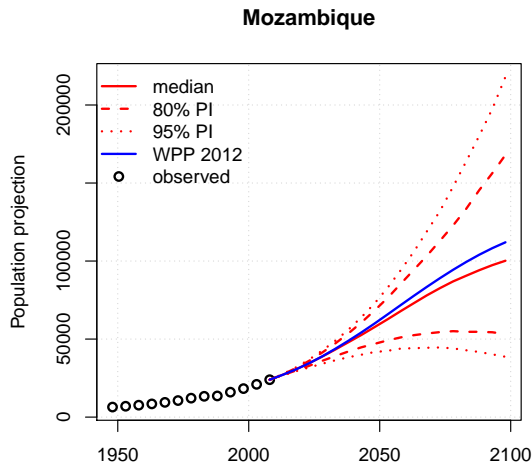


(c) Population Projection for Lesotho, female age 15-49

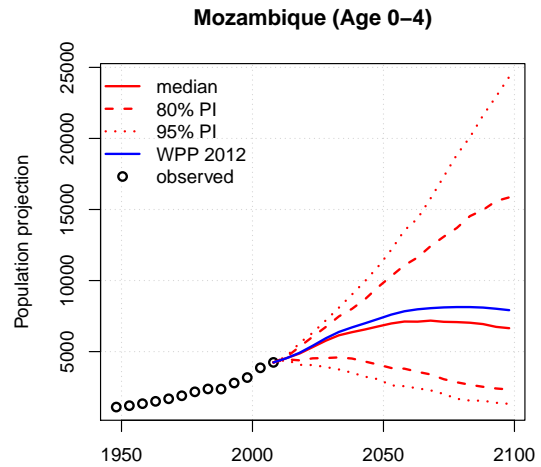


(d) Projection of the probability a female at age 10 will die before her 45th birthday

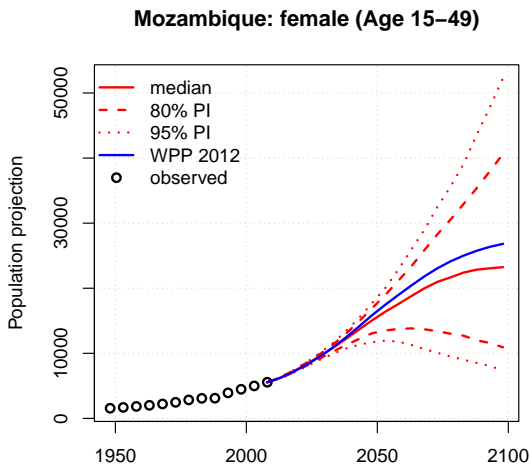
Figure 6: Probabilistic population projections for Lesotho 2010-2100. For panels a, b, and c: observed data: black circles; median probabilistic projection: solid red line, 80% predictive interval: dashed red lines; 95% predictive interval: dotted red lines; WPP 2012 Revision population projection: solid blue line.



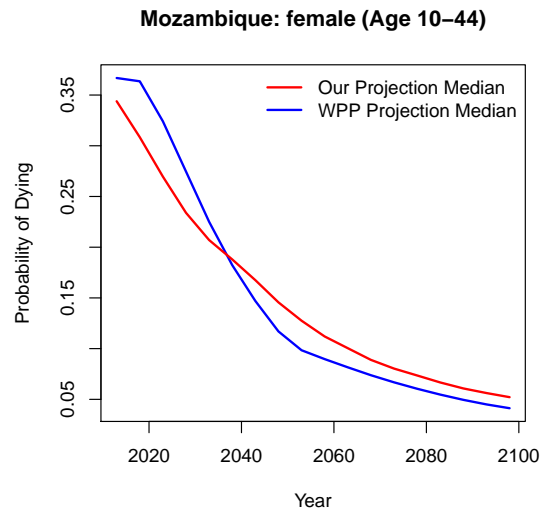
(a) Population Projection for Mozambique, total population



(b) Population Projection for Mozambique, age 0-4

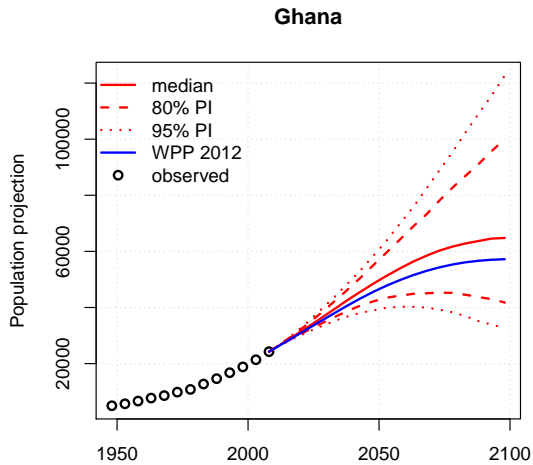


(c) Population Projection for Mozambique, female age 15-49

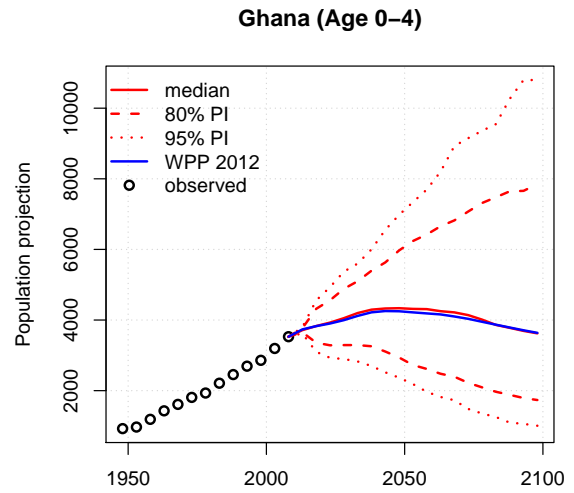


(d) Projection of the probability a female at age 10 will die before her 45th birthday

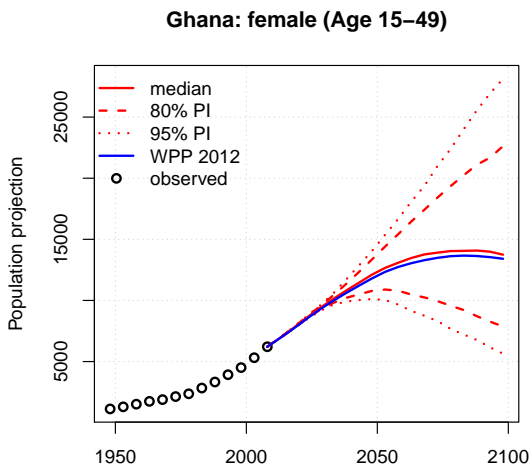
Figure 7: Probabilistic population projections for Mozambique 2010-2100. For panels a, b, and c: observed data: black circles; median probabilistic projection: solid red line, 80% predictive interval: dashed red lines; 95% predictive interval: dotted red lines; WPP 2012 Revision population projection: solid blue line.



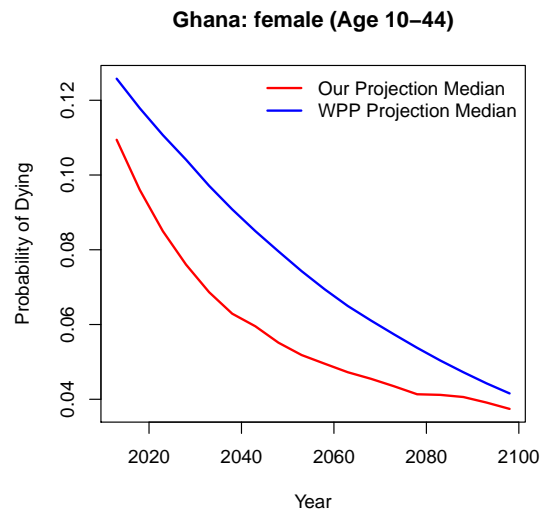
(a) Population Projection for Ghana, total population



(b) Population Projection for Ghana, age 0-4



(c) Population Projection for Ghana, female age 15-49



(d) Projection of the probability a female at age 10 will die before her 45th birthday

Figure 8: Probabilistic population projections for Ghana 2010-2100. For panels a, b, and c: observed data: black circles; median probabilistic projection: solid red line, 80% predictive interval: dashed red lines; 95% predictive interval: dotted red lines; WPP 2012 Revision population projection: solid blue line.